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δ-kicked Landau levels

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Abstract. We discuss the explicit analytical solution for the motion of a charged quantum particle in a homogeneous magnetic field under the influence of a series of δ-kicks of a cyclotron frequency. Transition probabilities between the Landau levels due to kicking are expressed in terms of the Jacobi polynomials whose arguments contain the reflection coefficient from a series of δ-walls.

1. Introduction

The Schrödinger equation for a charged particle moving in a magnetic field, uniform in space but varying in time, was solved in [1–4] in the frame of the method of time-dependent quantum integrals of motion. In particular, the generalizations of the Fock–Landau stationary solutions, as well as the amplitudes and probabilities of transitions between the energy eigenstates (the Landau levels), were found both for the circular gauge of the vector potential $A = \frac{1}{2}[H \times r]$ [2, 3] and for the Landau gauge [4]. Among the other results obtained in [2–4] we would like to mention the exact propagators for a charged non-stationary oscillator and a free particle in uniform time-dependent electric and magnetic fields. Also, the time-dependent coherent states were constructed. They generalize the stationary coherent states of an oscillator, introduced by Glauber [5], and those of a particle in a constant magnetic field, introduced in [6] (see also [7]). The other types of quantum states for a charge in a magnetic field, such as squeezed and correlated states, have been studied in [8–15]. The case of the relativistic (Klein–Gordon or Dirac) equations was considered in [16, 17]. An extensive list of other references to the papers devoted to different aspects of charged particle motion in magnetic and electric fields can be found in [18].

In this paper we apply the methods elaborated in [2–4] (and exposed in detail, for example, in [19, 20]) to give the exact explicit formulae (suitable for numerical analysis) for the transition probabilities between the energy levels when the magnetic field varies in time in the form of very short pulses. Besides deriving the general relations valid for any non-stationary magnetic field, we consider the special time dependence of the squared cyclotron frequency in the form of periodic δ-pulses in time. There are two reasons for this choice. First, it enables us to obtain simple explicit expressions which can be analysed in detail. Second, it is known [21, 22] that 'δ-kicked' systems can exhibit chaotic behaviour

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under certain conditions. Therefore it is interesting to investigate the transition probabilities between the Landau levels in the case when a 'δ-kicked' system admits exact solutions. In section 2 we show (following [2, 3]) the expressions for the integrals of motion and coherent states of a charge placed in the magnetic field of a solenoid which is uniform in space, but varying in time. In section 3 we give a generic expression for the probability distribution over the Landau levels of a charged particle subjected to a parametric excitation caused by pulses of the magnetic field. In section 4 we concentrate on the specific case of very short pulses modelled by a series of the periodic δ-kicks. Section 5 is devoted to the numerical analysis of the transition probabilities in the various special cases, and to the discussion of numerous two- and three-dimensional plots.

2. Integrals of motion and coherent states

Consider a particle with unit mass and charge placed in a classical electromagnetic field with the vector potential

$$A(t) = \tfrac{1}{2}[\boldsymbol{H}(t) \times \boldsymbol{r}] \tag{1}$$

where $\boldsymbol{r}$ is a vector defining the coordinates of the particle, and $\boldsymbol{H}(t)$ is a homogeneous time-dependent axially symmetric magnetic field. Although the scalar potential is supposed to be equal to zero, the charge is affected not only by the varying magnetic field, but also by the electric field $\boldsymbol{E}(t) = -\frac{1}{2}[\dot{\boldsymbol{H}}(t) \times \boldsymbol{r}]$ (we assume $c = \hbar = 1$).

We suppose that the magnetic field vector $\boldsymbol{H}$ is directed along the z-axis. Ignoring the trivial component of the motion in this direction, we only treat the motion in the (x, y)-plane, described by the Hamiltonian

$$\hat{H} = \tfrac{1}{2}\left[(\hat{p}_x + \omega(t)\hat{y})^2 + (\hat{p}_y - \omega(t)\hat{x})^2\right] \tag{2}$$

where $\omega(t) \equiv \frac{1}{2}H(t)$ is the *Larmor frequency*. The spin-dependent part of the Hamiltonian is not considered here.

By direct calculation it can be verified that the following Schrödinger operators commute with the operator $(\hat{H} - \mathrm{i}\partial/\partial t)$, and hence are integrals of motion:

$$\hat{A}(t) = \tfrac{1}{2}[\varepsilon(t)(\hat{p}_x + \mathrm{i}\hat{p}_y) - \mathrm{i}\dot{\varepsilon}(t)(\hat{y} - \mathrm{i}\hat{x})]\exp\left[\mathrm{i}\int^t \omega(\tau)\,\mathrm{d}\tau\right] \tag{3}$$

$$\hat{B}(t) = \tfrac{1}{2}[\varepsilon(t)(\hat{p}_y + \mathrm{i}\hat{p}_x) - \mathrm{i}\dot{\varepsilon}(t)(\hat{x} - \mathrm{i}\hat{y})]\exp\left[-\mathrm{i}\int^t \omega(\tau)\,\mathrm{d}\tau\right] \tag{4}$$

where $\varepsilon(t)$ is any particular solution of the equation

$$\ddot{\varepsilon} + \omega^2(t)\varepsilon = 0\,. \tag{5}$$

In order to get time-independent commutation relations of the operators $\hat{A}$, $\hat{A}^+$ and $\hat{B}$, $\hat{B}^+$ we choose the special solution of (5)

$$\varepsilon(t) = |\varepsilon|\exp\left[\mathrm{i}\int^t |\varepsilon(\tau)|^{-2}\,\mathrm{d}\tau\right]. \tag{6}$$

Then the modulus of the ε-function must obey the equation

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}|\varepsilon| + \omega^2(t)|\varepsilon| - |\varepsilon|^{-3} = 0\,. \tag{7}$$

The following commutation relations hold:

$$[\hat{A}, \hat{A}^+] = [\hat{B}, \hat{B}^+] = 1 \qquad [\hat{A}, \hat{B}] = [\hat{A}, \hat{B}^+] = 0\,.$$

If the Larmor frequency assumes the constant value ω_{in} when $t \to -\infty$, then we may choose the 'initial solution' of (5) in the form

$$\varepsilon_{in}(t) = \omega_{in}^{-1/2} \exp(\mathrm{i}\omega_{in} t) \qquad \dot{\varepsilon}_{in}(t) = \mathrm{i}\omega_{in}\varepsilon_{in}(t) . \tag{8}$$

The corresponding integrals of motion read as

$$\hat{A}_{in} = \omega_{in}^{1/2}[\hat{y} - \hat{y}_0 - \mathrm{i}(\hat{x} - \hat{x}_0)] \exp(2\mathrm{i}\omega_{in} t) \tag{9}$$

$$\hat{B}_{in} = \omega_{in}^{1/2}(\hat{x}_0 - \mathrm{i}\hat{y}_0) \tag{10}$$

where the operators

$$\hat{x}_0 = \frac{\hat{x}}{2} + \frac{\hat{p}_y}{2\omega_{in}} \qquad \hat{y}_0 = \frac{\hat{y}}{2} - \frac{\hat{p}_x}{2\omega_{in}}$$

are the well known centre-orbit coordinates of the particle moving in a constant magnetic field. From (9) and (10) it is evident that the eigenvalues of $\hat{B}_{in}$ define the coordinates of the orbit centre in the (x, y)-plane, while the eigenvalues of the operator $\hat{A}_{in} \exp(-2\mathrm{i}\omega_{in} t)$ give the relative coordinates of the particle with respect to the centre.

The coherent state $|\alpha, \beta; t\rangle$ is defined as the eigenstate of the time-dependent invariants $\hat{A}(t)$ and $\hat{B}(t)$:

$$\hat{A}|\alpha, \beta; t\rangle = \alpha|\alpha, \beta; t\rangle \qquad \hat{B}|\alpha, \beta; t\rangle = \beta|\alpha, \beta; t\rangle .$$

It can be represented as

$$|\alpha, \beta; t\rangle = \exp\left[-\tfrac{1}{2}|\alpha|^2 + |\beta|^2\right] \sum_{n_1,n_2=0}^{\infty} \frac{\alpha^{n_1}\beta^{n_2}}{\sqrt{n_1!\,n_2!}} |n_1, n_2; t\rangle \tag{11}$$

where α, β are arbitrary complex numbers, and

$$|n_1, n_2; t\rangle = \frac{(\hat{A}^+)^{n_1}(\hat{B}^+)^{n_2}}{\sqrt{n_1!\,n_2!}} |0, 0; t\rangle \tag{12}$$

stands for the time-dependent Fock state.

Using the commuting unitary displacement operators

$$\hat{D}(\alpha) = \exp(\alpha\hat{A}^+ - \alpha^*\hat{A}) \qquad \hat{D}^{-1}(\alpha)\hat{A}\hat{D}(\alpha) = \hat{A} + \alpha$$
$$\hat{D}(\beta) = \exp(\beta\hat{B}^+ - \beta^*\hat{B}) \qquad \hat{D}^{-1}(\beta)\hat{B}\hat{D}(\beta) = \hat{B} + \beta$$

on the ground state

$$\langle x, y|0, 0; t\rangle = \left(\pi\varepsilon^2\right)^{-1/2} \exp\left(\mathrm{i}\frac{\dot{\varepsilon}}{2\varepsilon}\left(x^2 + y^2\right)\right) \tag{13}$$

we get the explicit form of the coherent state wavefunction in the coordinate representation

$$\begin{aligned}\langle x, y|\alpha, \beta; t\rangle &= \langle x, y|\hat{D}(\alpha)\hat{D}(\beta)|0, 0; t\rangle \\ &= \left(\pi\varepsilon^2\right)^{-1/2} \exp\left(\mathrm{i}\frac{\dot{\varepsilon}}{2\varepsilon}\left(x^2 + y^2\right)\right) \\ &\quad \times \exp\left[-\tfrac{1}{2}\left(|\alpha|^2 + |\beta|^2\right) + \frac{1}{|\varepsilon|}\left(\beta(x + \mathrm{i}y)\mathrm{e}^{-\mathrm{i}\gamma_-} + \mathrm{i}\alpha(x - \mathrm{i}y)\mathrm{e}^{-\mathrm{i}\gamma_+}\right)\right] \\ &\quad \times \exp\left(-\mathrm{i}\alpha\beta\mathrm{e}^{-\mathrm{i}(\gamma_+ + \gamma_-)}\right)\end{aligned} \tag{14}$$

where

$$\gamma_\pm = \int^t \left[|\varepsilon|^{-2} \pm \omega(\tau)\right] \mathrm{d}\tau . \tag{15}$$

3. Transition probabilities

Let us take the magnetic field to be constant in the remote past and the remote future. More precisely, let us suppose

$$\omega(t) = \omega_{in} \qquad \text{if} \quad t < 0$$
$$\omega(t) = \omega_f \qquad \text{if} \quad t \to \infty .$$

Under these conditions, as $t \to \pm\infty$ there exist initial and final coherent states, as well as the initial and final Landau states. The transition amplitude connecting an initial state $|in\rangle$ with a final state $|f\rangle$ is given by the matrix element $T_{in}^f = \langle f|t \to \infty\rangle$, where $|t \to \infty\rangle$ stands for the $t \to \infty$ limit of the state which coincided with the $|in\rangle$-state when $t \to -\infty$. It is convenient to define the 'final' operators as

$$\hat{A}_f = (4\omega_f)^{-1/2}[\hat{p}_x + \mathrm{i}\hat{p}_y + \omega_f(\hat{y} - \mathrm{i}\hat{x})]\exp[\mathrm{i}(2\omega_f t + \varphi_0)] \tag{16}$$

$$\hat{B}_f = (4\omega_f)^{-1/2}\exp(-\mathrm{i}\varphi_0)[\hat{p}_y + \mathrm{i}\hat{p}_x + \omega_f(\hat{x} - \mathrm{i}\hat{y})] \tag{17}$$

where the additional phase equals

$$\varphi_0 = \int_0^\infty [\omega(t) - \omega_f]\,\mathrm{d}t .$$

The coherent states related to the constant field $\boldsymbol{H}_f$ are given in Cartesian coordinates by

$$\langle x, y|\gamma, \mu; f\rangle = \left(\frac{\omega_f}{\pi}\right)^{1/2} \exp\left[-\tfrac{1}{2}\left(2\mathrm{i}\omega_f t + \omega_f\left(x^2 + y^2\right) + |\gamma|^2 + |\mu|^2\right)\right]$$
$$\times \exp\left[\omega_f^{1/2}\left(\mu(x + \mathrm{i}y)\mathrm{e}^{\mathrm{i}\varphi_0} + \mathrm{i}\gamma(x - \mathrm{i}y)\mathrm{e}^{-\mathrm{i}(2\omega_f t + \varphi_0)}\right) - \mathrm{i}\gamma\mu\mathrm{e}^{-2\mathrm{i}\omega_f t}\right]. \tag{18}$$

For the Fock–Landau states a simple expression can be written in polar coordinates (φ, ρ)

$$\langle \rho, \varphi|m_1, m_2; f\rangle = \mathrm{i}^{m_1}(-1)^p \left(\frac{\omega_f p!}{\pi q!}\right)^{1/2} \exp\left[\mathrm{i}(m_2 - m_1)(\varphi + \varphi_0)\right]$$
$$\times \exp\left[-\tfrac{1}{2}\omega_f\rho^2 - \mathrm{i}\,(2m_1 + 1)\,\omega_f t\right]\left(\omega_f\rho^2\right)^{|m_1 - m_2|/2} L_p^{|m_1 - m_2|}\left(\omega_f\rho^2\right) \tag{19}$$

where $p = \min(m_1, m_2)$, $q = \max(m_1, m_2)$, and $L_p^m(z)$ means the associated Laguerre polynomial.

Comparing (16) and (17) with (3) and (4) we get the linear relations

$$\hat{A}(t) = \mathrm{e}^{-\mathrm{i}\nu}\left(\xi\hat{A}_f + \eta\hat{B}_f^+\right) \tag{20}$$

$$\hat{B}(t) = \mathrm{e}^{\mathrm{i}\nu}\left(\eta\hat{A}_f^+ + \xi\hat{B}_f\right) \tag{21}$$

where

$$\xi(t) = (4\omega_f)^{-1/2}(\omega_f\varepsilon - \mathrm{i}\dot{\varepsilon})\exp[-\mathrm{i}\omega_f t] \tag{22}$$

$$\eta(t) = (4\omega_f)^{-1/2}(\mathrm{i}\omega_f\varepsilon - \dot{\varepsilon})\exp[\mathrm{i}\omega_f t] \tag{23}$$

$$\nu(t) = \int_t^\infty [\omega(\tau) - \omega_f]\,\mathrm{d}\tau . \tag{24}$$

The choice of the ε-function in (6) results in the identity

$$|\xi|^2 - |\eta|^2 \equiv 1 . \tag{25}$$

In the limit of $t \to \infty$ (or when the magnetic field takes the final constant value after time T, as in the special case considered in the next section) $\nu = 0$, so that ξ and η become constant coefficients related to the ε-function as follows:

$$\varepsilon_f(t) = \omega_f^{-1/2}[\xi\exp(\mathrm{i}\omega_f t) - \mathrm{i}\eta\exp(-\mathrm{i}\omega_f t)] . \tag{26}$$

Since the solutions of the Schrödinger equation are given in terms of the function $\varepsilon(t)$, all the transition amplitudes are completely determined by the parameters ξ and η.

For the transition amplitude between coherent states $|\alpha, \beta; in\rangle$ and $|\gamma, \mu; f\rangle$ we get

$$T_{\alpha,\beta}^{\gamma,\mu} = \xi^{-1} \exp\left[-\tfrac{1}{2}\left(|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\mu|^2\right) + \xi^{-1}(\alpha\beta\eta^* + \beta\mu^* + \alpha\gamma^* - \gamma^*\mu^*\eta)\right]. \tag{27}$$

Using this formula and the property that the coherent state is the generating function for number states one can find the transition amplitudes between the energy and angular momentum eigenstates:

(i)

$$T_{n_1 n_2}^{m_1 m_2} = \frac{(-1)^{m_1-n_1}}{\xi}\left(\frac{n_1!m_2!}{n_2!m_1!}\right)^{1/2}\frac{(\xi^*)^{n_1}\eta^{m_1-n_1}}{\xi^{m_2}}P_{n_1}^{(m_1-n_1,n_2-n_1)}\left(1-2\frac{|\eta|^2}{|\xi|^2}\right) \tag{28}$$

when $n_2 \geqslant n_1$, $m_i \geqslant n_i$, $i = 1, 2$;

(ii)

$$T_{n_1 n_2}^{m_1 m_2} = \frac{1}{\xi}\left(\frac{n_2!m_1!}{m_2!n_1!}\right)^{1/2}\frac{(\xi^*)^{m_1}(\eta^*)^{n_1-m_1}}{\xi^{n_2}}P_{m_1}^{(n_1-m_1,n_2-n_1)}\left(1-2\frac{|\eta|^2}{|\xi|^2}\right) \tag{29}$$

when $n_2 \geqslant n_1$, $m_i < n_i$, $i = 1, 2$.

Here $P_n^{(\alpha,\beta)}(x)$ means the Jacobi polynomial. The initial and final quantum numbers must satisfy the relation $m_2 - m_1 = n_2 - n_1$, which is merely the conservation of the angular momentum L_z. Both (28) and (29) are related to the positive values $L_z = n_2 - n_1 \geqslant 0$. In the case of negative L_z one should make the replacement of indices $1 \leftrightarrow 2$. If we did not introduce the phase φ_0 in the definition of the final states (18) and (19), it would appear in the transition amplitudes.

The transition probability in case (i) reads

$$W_{n_1 n_2}^{m_1 m_2} = \frac{m_2!n_1!}{m_1!n_2!}R^{m_1-n_1}(1-R)^{n_2-n_1+1}|P_{n_1}^{(m_1-n_1,n_2-n_1)}(1-2R)|^2 \tag{30}$$

where

$$R = \frac{|\eta|^2}{|\xi|^2} < 1. \tag{31}$$

In case (ii) one should change $n_i \leftrightarrow m_i$, $i = 1, 2$.

We see that the transition probabilities between all Landau levels are completely determined by the probability of remaining in the ground state

$$W_{00}^{00} = 1 - R.$$

The probabilities do not depend on the sign of L_z:

$$W_{n_1 n_2}^{m_1 m_2} = W_{n_2 n_1}^{m_2 m_1}.$$

Moreover,

$$W_{n_1 n_2}^{m_1 m_2} = W_{m_1 m_2}^{n_1 n_2}. \tag{32}$$

From the general principles of quantum mechanics it follows that the last relation must hold for any time-reversible Hamiltonian. The Hamiltonian (2) is not time-reversible in the general case, nonetheless (32) is valid. One can easily understand this fact if it is taken into account that (5) may be treated as the one-dimensional stationary Schrödinger equation

$$\psi'' + 2[E - U(x)]\psi = 0$$

provided time t is replaced by the coordinate x, and the effective potential $U(x)$ is given by

$$\omega^2(x) = \omega_{in}^2 + 2[U(-\infty) - U(x)] \qquad E = \tfrac{1}{2}\omega_{in}^2 + U(-\infty)\,.$$

Then parameter R (see (31)) can be interpreted as the probability reflection coefficient of a particle moving through the potential $U(x)$, and (32) is a consequence of the known property of this coefficient that it is the same for waves moving from the left or the right.

4. δ-kicking

We now proceed with calculating the transition probabilities between the Landau levels caused by a set of N δ-pulses of the magnetic field, when the Larmor frequency varies in time as

$$\omega^2(t) = \omega_0^2\left[1 + 2\kappa\omega_0^{-1}\sum_{j=0}^{N-1}\delta(t - jT)\right] \tag{33}$$

where $\kappa \geqslant 0$ is a dimensionless parameter characterizing the strength of kicks. In this specific case the initial Larmor frequency coincides with the final one: $\omega_{in} = \omega_f = \omega_0$. The effective potential introduced at the end of the preceding section reads

$$U(x) = -\kappa\omega_0\sum_{j=0}^{N-1}\delta(x - jT) \tag{34}$$

and the effective wavenumber of a particle moving through this potential *well* equals ω_0. After the lth kick the solution of (5) can be represented as (see (26))

$$\varepsilon_l(t) = \omega_0^{-1/2}[\xi_l\exp(\mathrm{i}\omega_0 t) - \mathrm{i}\eta_l\exp(-\mathrm{i}\omega_0 t)]$$

with $\xi_0 = 1$ and $\eta_0 = 0$. The matching conditions for the ε-function and its derivatives yield the following recurrence relations between the coefficients $\tilde{\xi}_l = \xi_l\exp(\mathrm{i}\omega_0 lT)$ and $\tilde{\eta}_l = \eta_l\exp(-\mathrm{i}\omega_0 lT)$ before and after every kick:

$$\left\|\begin{matrix}\tilde{\xi}_{l+1}\\ \tilde{\eta}_{l+1}\end{matrix}\right\| = \mathbf{S}\circ\left\|\begin{matrix}\tilde{\xi}_l\\ \tilde{\eta}_l\end{matrix}\right\|$$

where

$$\mathbf{S} = \left\|\begin{matrix}(1+\mathrm{i}\kappa)\mathrm{e}^{si\Phi} & \kappa\mathrm{e}^{\mathrm{i}\Phi}\\ \kappa\mathrm{e}^{-\mathrm{i}\Phi} & (1-\mathrm{i}\kappa)\mathrm{e}^{-\mathrm{i}\Phi}\end{matrix}\right\| \qquad \Phi = \omega_0 T\,.$$

In the case of the single kick $(N = 1)$ we get

$$\xi_1 = 1 + \mathrm{i}\kappa \qquad \eta_1 = \kappa$$

so that the reflection coefficient is a simple monotonous function of the kick strength

$$R_1(\kappa) = \frac{\kappa^2}{1+\kappa^2}\,. \tag{35}$$

A more interesting behaviour of this coefficient is observed for the multiple kicks. Then

$$\left\|\begin{matrix}\tilde{\xi}_N\\ \tilde{\eta}_N\end{matrix}\right\| = \mathbf{S}^N\circ\left\|\begin{matrix}\tilde{\xi}_0\\ \tilde{\eta}_0\end{matrix}\right\|$$

and since matrix $\mathbf{S}$ is unimodular, $\det\mathbf{S} = 1$, its powers can be expressed as (see, for example, [23, 24])

$$\mathbf{S}^N = U_{N-1}\big(\tfrac{1}{2}\mathrm{Tr}\,\mathrm{S}\big)\circ\mathbf{S} - U_{N-2}\big(\tfrac{1}{2}\mathrm{Tr}\,\mathbf{S}\big)\circ\mathbf{I} \tag{36}$$

where $\mathbf{I}$ means the unit matrix, and $U_n(x)$ is the Chebyshev polynomial of the second kind,

$$U_n(\cos\theta) = \frac{\sin(n+1)\theta}{\sin\theta}.$$

Thus we have

$$\xi_N = (1+\mathrm{i}\kappa)\mathrm{e}^{-\mathrm{i}(N-1)\Phi}U_{N-1}(\chi) - \mathrm{e}^{-\mathrm{i}N\Phi}U_{N-2}(\chi)$$
$$\eta_N = \kappa \mathrm{e}^{\mathrm{i}(N-1)\Phi}U_{N-1}(\chi)$$

where

$$\chi = \cos\Phi - \kappa\sin\Phi. \tag{37}$$

The reflection coefficient reads (we take into account the identity (25))

$$R_N(\kappa,\Phi) = \frac{\kappa^2[U_{N-1}(\cos\Phi - \kappa\sin\Phi)]^2}{1+\kappa^2[U_{N-1}(\cos\Phi - \kappa\sin\Phi)]^2}. \tag{38}$$

If $\Phi = \pi m$ with $m = 1, 2, 3, \ldots$, then the response of the system to N δ-kicks is equivalent to its reaction to a single impulse with an amplitude multiplied by N, because $U_n(1) = n+1$. But for other relations between the Larmor frequency and the period of kicking the reflection coefficient may be a highly oscillating function of κ, since the nth Chebyshev polynomial has n zeros in the interval $-1 < \chi < 1$. From the definition of the polynomial $U_{N-1}(\cos\theta)$ we see that its zeros correspond to the values $\chi_0^{(m)} = \cos(\pi m/N)$ with $m = 1, 2, \ldots, N-1$. Suppose for definiteness that $\sin\Phi > 0$ and $\cos\Phi \geqslant 0$. Then the reflection coefficient turns into zero for $\kappa_0^{(m)} = [\cos(\Phi) - \cos(\pi m/N)]/\sin\Phi$ (more precisely, for those values of m that yield $\kappa_0^{(m)} > 0$, since the parameter κ is assumed to be non-negative). If the number of kicks N is sufficiently large, then the value of κ changes only slightly when m is replaced by $m+1$. In such a case the local maxima of the function $R(\kappa)$ are located near the points $\kappa_{\max}^{(m)} = \{\cos(\Phi) - \cos(\pi[2m+1]/2N)\}/\sin\Phi$ with $m = 1, 2, \ldots, N-2$. Therefore at $N \gg 1$ the function $R(\kappa)$ exhibits strong oscillations with the envelope

$$R_{\max}(\kappa) = \frac{\kappa^2}{(\kappa\cos\Phi + \sin\Phi)^2}. \tag{39}$$

Such a picture is observed until $\kappa \approx \kappa_* = \cot(\Phi/2)$. In the vicinity of the point κ_* the behaviour of the curve $R(\kappa)$ changes rapidly, and it approaches the asymptotic line $R = 1$. For instance, if $\Phi = \pi/2$, then the transition point is $\kappa_* = 1$, and $R(\kappa)$ exhibits approximately $N/2$ oscillations with a simple parabolic envelope $R_{\max} = \kappa^2$. But for $\Phi \ll 1$ the transition point goes far into the region of large kick magnitudes: $\kappa_* \approx 2/\Phi \gg 1$. In this case approximately N oscillations can be observed, and the envelope $R_{\max}(\kappa) \approx \kappa^2/(\kappa+\Phi)^2$ practically coincides with the asymptotic line almost from the beginning. It is interesting to note some instability in the behaviour of the function $R_N(\kappa,\Phi)$ at the points $\Phi = \pi m$. For example, if $\Phi = \pi - \epsilon$, then $\chi \approx -1 - \epsilon\kappa$, so that the reflection coefficient exhibits no oscillations when κ increases from zero. However, for $\Phi = \pi + \epsilon$ we have $\chi \approx -1 + \epsilon\kappa$, and strong oscillations are observed.

5. Influence of kicks on the occupation of Landau levels

This section is devoted to the 'visualization' of the formulae given above. Let us begin with (30). In figure 1 we plot the values of $W_{00}^{m_1 m_1}(R)$ versus $m_1 \in \{0, 10\}$ and $R \in \{0, 1\}$. In this case the absolute maximum of probabilities is achieved for small values of m_1 and R. As soon as m_1 increases the maximum shifts to the right in the R-increasing direction. This is seen distinctly in figure 2, where a two-dimensional graph of $W_{00}^{55}(R)$ is reported.

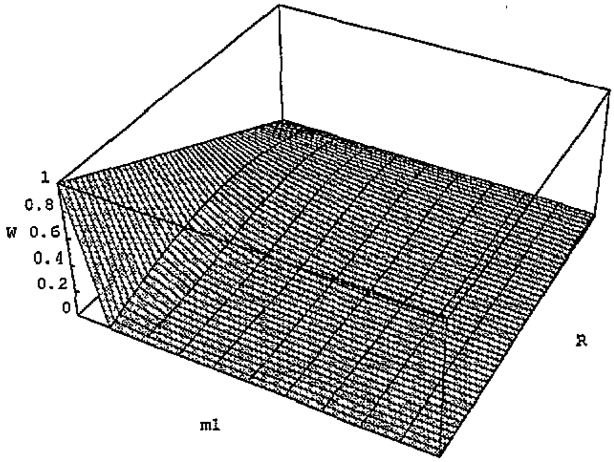

Figure 1. $W_{00}^{m_1 m_1}(R)$ with $m_1 \in \{0, 10\}$, $R \in \{0, 1\}$.

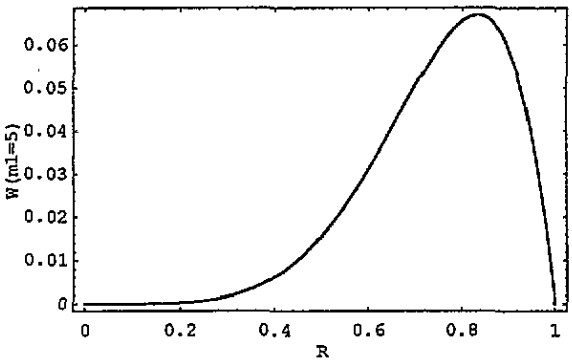

Figure 2. Two-dimensional plot of $W_{00}^{55}(R)$ with $R \in \{0, 1\}$.

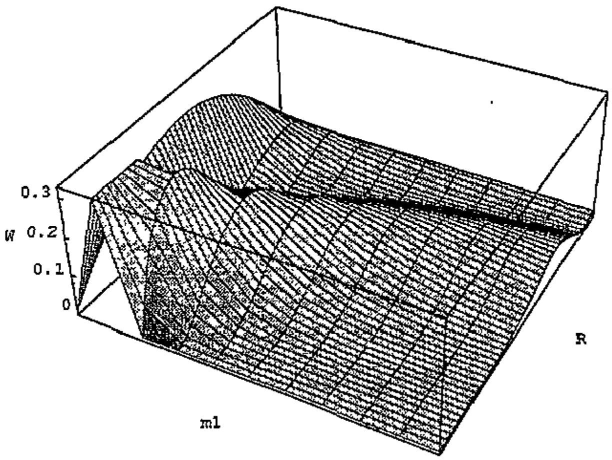

Figure 3. $W_{11}^{m_1 m_1}(R)$ with $m_1 \in \{0, 10\}$, $R \in \{0, 1\}$.

When we move from the initial state $|0, 0\rangle$ to higher states the plots become more interesting. For instance, the plot of $W_{11}^{m_1 m_1}$ shown in figure 3 has a richer structure:

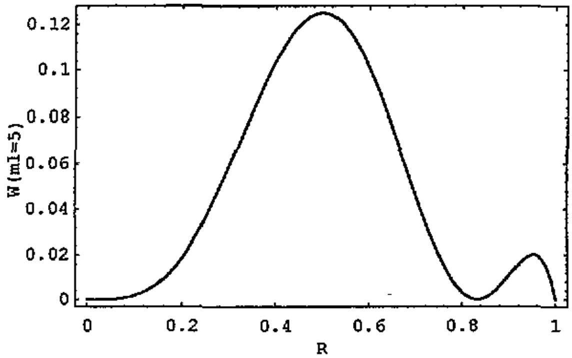

Figure 4. Two-dimensional plot of $W_{11}^{55}(R)$ with $R \in \{0, 1\}$.

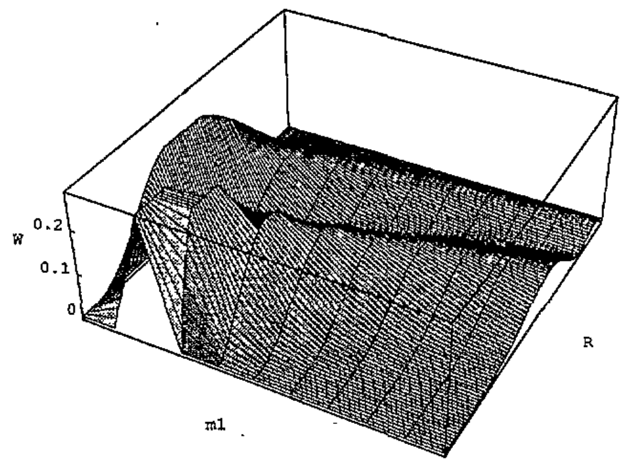

Figure 5. $W_{21}^{m_1,(m_1-1)}(R)$ with $m_1 \in \{0, 10\}$, $R \in \{0, 1\}$.

although the absolute maximum for the probabilities is still at $R \sim 0$, it now corresponds to $m_1 = 1$. When m_1 increases this maximum moves to $R = 1$. Moreover, the second maximum arises, as shown in figure 4. Since the Jacobi polynomial has zeros when its argument is within to the interval $(-1, 1)$, it is not surprising that the probabilities become zero for some values of R different from 0 and 1. For example,

$$W_{11}^{m_1m_1}(R) = \tfrac{1}{4}(1-R)R^{m_1-1}(m_1 - 1 + (m_1+1)(1-2R))^2$$

and this function has three zeros $R = 0$, $R = 1$, $R = m_1/(m_1 + 1)$.

In figure 5 we show the behaviour of $W_{2\,1}^{m_1,(m_1-1)}(R)$ for $m_1 \in \{0, 10\}$, $R \in \{0, 1\}$. The two maxima get closer in amplitude, as one can verify comparing figures 6 and 4. This time the zeros for $W_{2\,1}^{m_1,(m_1-1)}(R)$ are $R = 0$, $R = 1$, and $R = (m_1 - 1)/(m_1 + 1)$.

Figures 7 and 9 correspond to $W_{3\,2}^{m_1,(m_1-1)}$ and $W_{6\,3}^{m_1,(m_1-3)}(R)$, respectively. The zeros of $W_{3\,2}^{m_1,(m_1-1)}(R)$ (besides $R = 0$ and 1) are given by

$$R = \frac{m_1^2 - 1 \pm \sqrt{3(m_1^2 - 1)}}{m_1^2 + 3m_1 + 2}.$$

The two-dimensional graphs of $W_{32}^{54}(R)$ and $W_{63}^{74}(R)$ shown ir figures 8 and 10 demonstrate the presence of three and four distinct maxima, respectively.

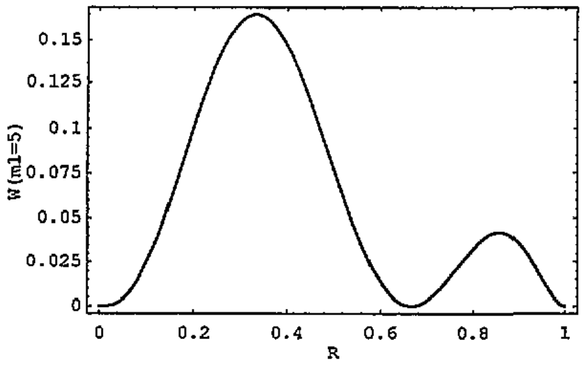

Figure 6. Two-dimensional plot of $W_{21}^{54}(R)$ with $R \in \{0, 1\}$.

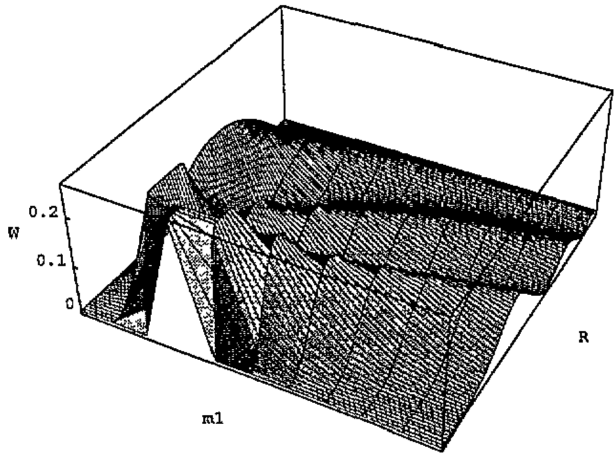

Figure 7. $W_{32}^{m_1,(m_1-1)}(R)$ with $m_1 \in \{0, 10\}$, $R \in \{0, 1\}$.

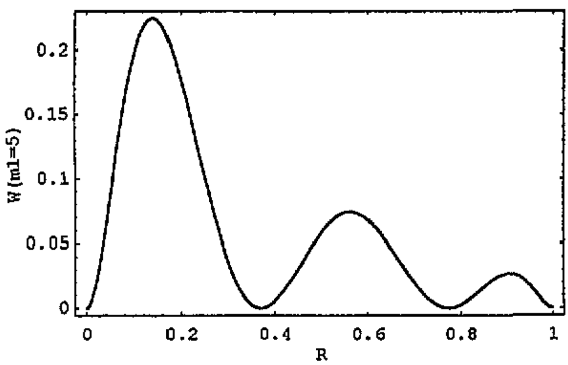

Figure 8. Two-dimensional plot of $W_{32}^{54}(R)$ with $R \in \{0, 1\}$.

Finally figures 11(a) and (b) show the behaviour of R versus κ as a function of η and ξ: $R = |\eta/\xi|^2$. In figure 11(a) we let $\kappa \in \{0, 2\}$ because, in this case, R very rapidly reaches the asymptotic value $R = 1$. In figure 11(b) the behaviour of R is more interesting.

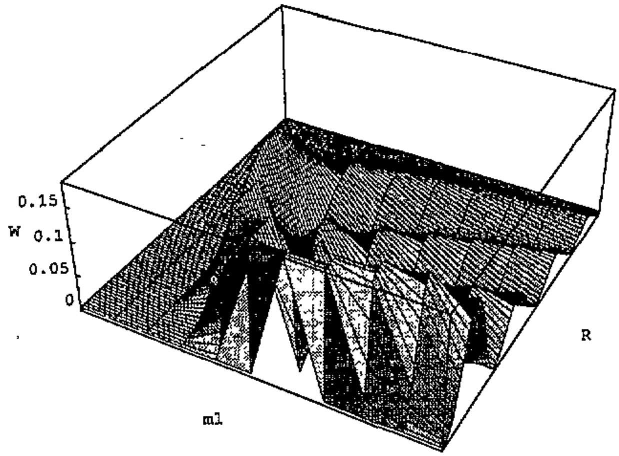

Figure 9. $W_{63}^{m_1,(m_1-3)}(R)$ with $m_1 \in \{0, 10\}$, $R \in \{0, 1\}$.

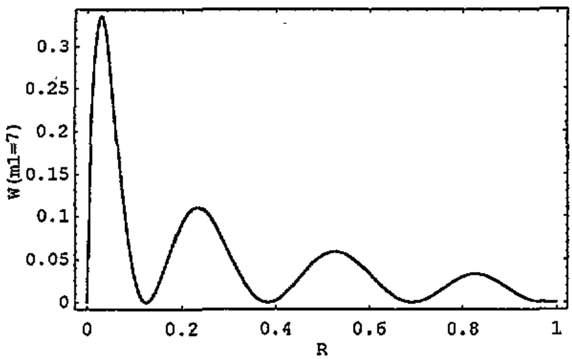

Figure 10. Two-dimensional plot of $W_{63}^{74}(R)$ with $R \in \{0, 1\}$.

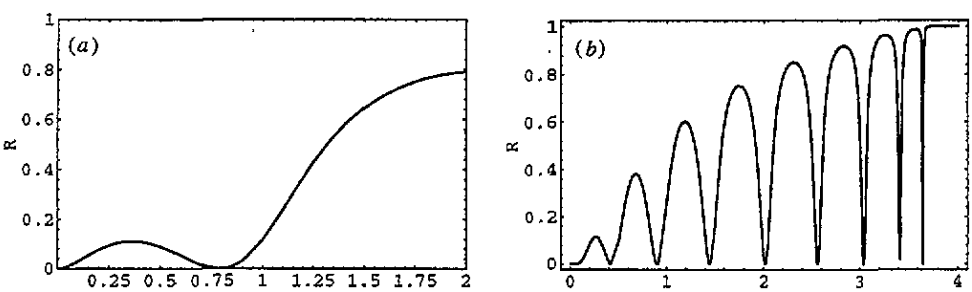

Figure 11. (*a*) Two-dimensional plot of $R = R(\kappa)$ with $\Phi = 1/2, N = 3, \kappa \in \{0, 2\}$. (*b*) Two-dimensional plot of $R = R(\kappa)$ with $\Phi = \pi/6, N = 11, \kappa \in \{0, 4\}$.

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